Structural Characterization on Index of DAEs in Hybrid Analysis for General Circuits

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1 Introduction In circuit simulation, we set up a system of equations by using circuit analysis methods such as the modified nodal analysis (MNA), the loop analysis, the cutset analysis, and the hybrid analysis. MNA is the most popular method, because it allows an automatic setup of model equations. In contrast, the hybrid analysis retains flexibility, which can be exploited to find a model description that reduces the numerical difficulties.

Circuit analysis methods lead to *differential-algebraic equations* (DAEs), which consist of algebraic equations and differential operations. The numerical difficulty of DAEs is measured by the *index*. In general, the higher the index is, the more difficult it is to solve the DAE. The index more than one is called the *higher index*. The difficulties of DAEs with higher index are much greater than DAEs with index zero or one. While many different concepts exist to assign an index to a DAE, we focus on the *tractability index* [3].

For nonlinear time-varying circuits which are composed of independent voltage/current sources, resistors, inductors, capacitors, and a wide class of dependent sources, Schwarz and Tischendorf [3] showed that the index of a DAE arising from MNA does not exceed two, and gave an index characterization. This result suggests that DAEs arising from MNA often have higher index.

The *hybrid analysis* is a common generalization of the loop analysis and the cutset analysis. Kron proposed the hybrid analysis in 1939, and Amari and Branin developed it further in 1960s. While the procedure of MNA is uniquely determined, the hybrid analysis starts with selecting a *partition* of elements and a *reference tree* in the network. This selection determines DAEs, called the *hybrid equations*, to be solved numerically. Thus it is natural to seek for an optimal selection that makes the hybrid equations easy to solve.

Recently, the analysis of the index of the hybrid equations has been developed. For linear time-invariant circuits which are composed of resistors, inductors, capacitors, independent voltage/current sources, and dependent voltage/current sources, an algorithm for finding an optimal hybrid analysis which minimizes the index of the hybrid equations is proposed in [1]. For nonlinear time-varying circuits which may contain a certain restricted class of dependent sources, it is proved in [2] that the index of the hybrid equations is at most one. A structural characterization of circuits with index zero is also given in [2]. From the practical point of view, however, it is important to deal with general dependent sources, which often result in higher index DAEs.

In this paper, extending the results in [2] to the circuits with general dependent sources, we give structural characterizations of circuits with index zero and at most one. This enables us to determine efficiently whether the hybrid equations of a circuit have higher index or not.

2 Nonlinear Time-Varying Circuits We consider nonlinear time-varying circuits composed of resistors, inductors L, capacitors C, independent voltage/current sources V and J, and dependent voltage/current sources S_U and S_I. We assume that inductors and capacitors are strictly passive, and resistors are reciprocal and strictly passive.

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Let $\Gamma = (W, E)$ be the network graph with vertex set W and edge set E. We denote the set of edges corresponding to independent voltage sources and independent current sources by E_v and E_j, respectively. We split $E \setminus (E_v \cup E_j)$ into E_y and E_z. A partition (E_y, E_z) is called an *admissible partition*, if E_y includes all the capacitors and all the dependent current sources, and E_z includes all the inductors and all the dependent voltage sources.

We call a spanning tree T of Γ a *normal reference tree* if T contains all edges in E_v, no edges in E_j, and as many edges as possible in the order corresponding to C, S_I, resistors in E_y, resistors in E_z, S_U, and L. The hybrid equations are determined by an admissible partition (E_y, E_z) and a normal reference tree T.

3 Index of Hybrid Equations This section gives a main theorem concerning the index of the hybrid equations. We now introduce the *Resistor-Acyclic condition* for admissible partition (E_y, E_z).

[Resistor-Acyclic condition]

- Each resistor in E_y and each dependent current source in S_I belong to a cycle consisting of independent voltage sources, capacitors, and itself.
- Each resistor in E_z and each dependent voltage source in S_U belong to a cutset consisting of inductors, independent current sources, and itself.

Let Γ° denote the graph obtained by contracting all edges in $V \cup C$ and deleting all edges in $L \cup J$. Then, we obtain the following necessary and sufficient conditions.

Theorem 1 *The index of the hybrid equations is zero if and only if the admissible partition (E_y, E_z) satisfies the Resistor-Acyclic condition.*

Theorem 2 *The index of the hybrid equations is at most one if and only if Γ° contains neither a cycle consisting of dependent voltage sources nor a cutset consisting of dependent current sources.*

Theorems 1 and 2, together with the results in [3], lead to the following theorem.

Theorem 3 *For nonlinear time-varying circuits composed of resistors, inductors, capacitors, and independent voltage/current sources, the minimum index of the hybrid equations never exceeds the index of a DAE arising from MNA.*

This suggests that the hybrid analysis may be superior to MNA in numerical accuracy.

References

[1] S. Iwata and M. Takamatsu, Index minimization of differential-algebraic equations in hybrid analysis for circuit simulation, *Mathematical Programming*, to appear.

[2] S. Iwata, M. Takamatsu, and C. Tischendorf, Hybrid analysis of nonlinear time-varying circuits providing DAEs with index at most one, *Proceedings of the 7th International Conference on Scientific Computing in Electrical Engineering*, to appear.

[3] D. E. Schwarz and C. Tischendorf, Structural analysis of electric circuits and consequences for MNA, *International Journal of Circuit Theory and Applications* (2000) **28**, 131–162.